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# Performance of Cascaded H-Bridge multilevel Inverters of Different Levels and with different Modulation Index

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**Abstract:** *The ordinary Voltage source inverter (VSI), whether it is single-phase or three-phase, has two output levels $+V_{dc}/2$ and $–V_{dc}/2$. This square waveform causes derating of the appliances or load devices as output voltage contains lower and higher order harmonics in addition to fundamental. For medium and high-power applications, multi-stage VSI (series and/or parallel combinations) with suitable filter (for obtaining near sinusoidal waveform at the output) may be a solution but there are problems in matching of different stages, selection of bulky transformer and proper filter.*
*The quality of both the output voltage and current waveforms with minimum ripple can be obtained using high frequency switching using various PWM techniques to modify the square wave output to modified square wave output. However, conventional two-level inverters have limitations in operating at high frequencies mainly due to switching losses in power semiconductor devices and constraints in device rating. The problem is resolved by the use of multilevel inverters (MLIs)*
*which utilize lower switching frequencies and give high voltages with improved total harmonic distortion (THD) without use of filter. This paper gives a brief review on Cascaded H-bridge Multi Level Inverters (CHMLI) techniques considering different number of levels. The main*
*disadvantages of MLI are requirement for isolated power supplies for each H-bridge, design complexity and switching control circuits for proper elimination of lower order harmonics for which mainly Selective Harmonic Elimination PWM technique is used as this technique has the ability to select low switching frequency of the power semiconductor devices to avoid high switching losses.*

***Keywords -****Harmonic Factor (HF), inverter, power semiconductor devices, SHE-PWM technique, simulation, total harmonic distortion (THD), transcendental equation.*

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## I. Introduction

A multilevel inverter has the ability that a desired output voltage can be synthesized with the use of several H-bridges which use dc voltages one for each H-bridge and connected in series. This inverter can be used at high voltage and high-power applications with lower dv/dt per switching and offers high efficiency and low electromagnetic interference [1],[9]. A simple H-bridge inverter circuit as shown in Fig. 1(a), the output voltage levels can be obtained as +Vdc, 0V and –Vdc as shown in Fig. 1(b), when it is oper4ated with a switching pattern as shown in Table I, and is termed as three-level inverter.

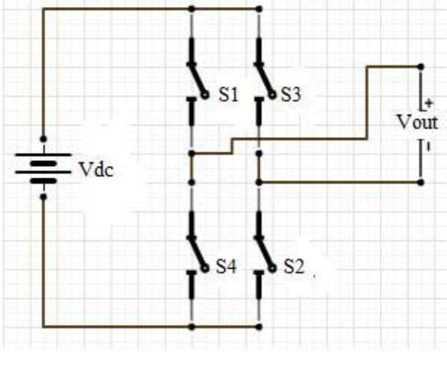


**Fig.1(a).** Circuit diagram of a three-level inverter

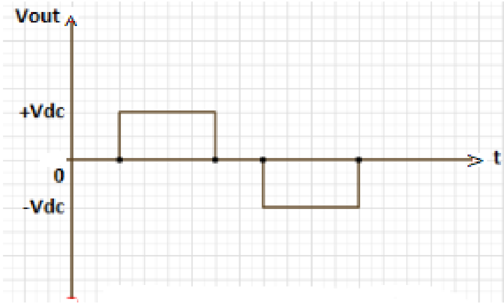


**Fig.1(b).** Voltage waveform of three-level inverter

**Table I** shows the switching of semiconductor devices S1, S2, S3 and S4.

| Switch Condition | Output |
|---|---|
| S1 and S2 closed | +Vdc |
| S3 and S4 closed | -Vdc |
| Either S1 and S3 or S2 and S4 are closed | OV |

The commonly used technique for generation of PWM as switching pulses is SHE-PWM (selective harmonic elimination pulse width modulation) where transcendental equations are formed after selecting the order of harmonics to be eliminated. For a (2S+1) multilevel inverters having S number of separate dc sources (considering fixed source voltages for simplicity), the number of harmonics which can be eliminated is (S-1) when each switch is made ON once during a single cycle of operation. To eliminate these (S-1) harmonics, S number of transcendental equations, which are nonlinear in nature, are to be solved to find out the switching angels with desired amplitude of fundamental for a given modulation index M, using suitable numerical methods [1],[2],[3],[10]. A complete comparison of 5-level , 7-level, 9-level and 11-level inverters with cascaded H-bridges using MATLAB-Simulink have been presented with computation of switching angels and analysis has been made for the harmonic voltages and total harmonic distortion (THD) for different values of modulation index M.

There are various numerical methods/algorithms to solve the above nonlinear equations to find out the switching angles. The Newton-Raphson (N-R) method is one of the fastest iterative methods which has been used here.

## II. Multilevel Inverter

Because of various constraints with two level inverters, the use of multilevel inverter technology is thought off that can produce an almost sinusoidal like output without much filtering. If the power semiconductor devices are switched through PWM controller then the output waveform will be PWM sinusoidal and the amplitude also can be controlled.

As mentioned earlier a cascaded H-bridge multilevel inverter consists of number of separate voltage source H-bridge connected in series as shown in fig.1(a). S1, S2, S3 and S4 can be any device for example, power BJT or MOSFET or IGBT or GTO[8],[9],[10]. Each H-bridge can produce three output voltage level, i.e., +Vdc, 0 and -Vdc. For a multilevel inverter with two H-bridges, the output voltage $V_0$ can be any of the five levels +2Vdc, Vdc, 0, -Vdc and -2Vdc. Each H-bridge operates with a switching sequence as shown in Table I.

For a (2S+1) multilevel inverter having S number of H-bridges as shown in Fig.2, using Fourier series, the general expression for the instantaneous voltage of the wave form can be expressed as [9]

$$V_0(t)=\frac{4V_{dc}}{n\pi}\sum_{n=1,3,5,7,....}^{\propto}[\cos(n\theta_1) + \cos(n\theta_2) + ....... + \cos(n\theta_S)]\sin(n\omega t) ...... (1)$$

Where $\theta_S$ is the switching angle of the Sth H-bridge and
$0 < \theta_1 < \theta_2 < \theta_3 .... < \theta_S < \pi/2$.

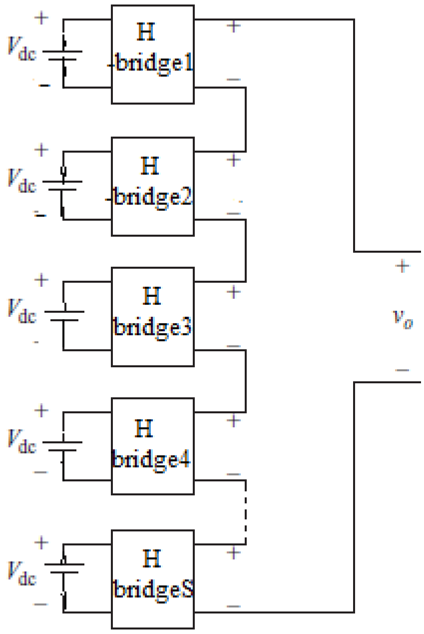


**Fig.2:** Multilevel Inverter with S no H-bridges.

The amplitude of nth harmonic is given by

$V_n = \frac{4V_{dc}}{n\pi}[\cos(n\theta_1) + \cos(n\theta_2) + \ldots\ldots + \cos(n\theta_S)] \ldots$ (2)

For n=1, 3, 5, 7….to ∞.

The amplitude of fundamental is given by

$V_1 = \frac{4V_{dc}}{\pi}\ [\cos(\theta_1) + \cos(\theta_2) + \ldots\ldots + \cos(\theta_S)] \ldots$.. (2a)

$V_{1,max} = \frac{4V_{dc}}{\pi} x\ s$

The modulation index M for S no of dc sources having voltage Vdc each is given by

$M = \frac{V_1}{V_{1,max}} = \frac{V_1}{4SVdc/\pi} = \frac{[\cos(\theta_1) + \cos(\theta_2) + \ldots\ldots + \cos(\theta_S)]}{S}$

Or, $[\cos(\theta_1) + \cos(\theta_2) + \ldots + \cos(\theta_S)] = SM$ …. (3)

Harmonic Factor (HF) is defined by,

Harmonic Factor, $HF = \frac{V_n}{V_1}$ . .... (3a)

Total Harmonic Distortion, $THD = \frac{(V_{o,rms}^2 - V_{1,rms}^2)^{\frac{1}{2}}}{V_{1,rms}}$, where

$\mathbf{V_{o,\,rms}}$ =RMS value of output voltage waveform, and

$V_{1,\,rms}$ = RMS value of fundamental of output voltage waveform.

From equation (2) it is obvious that to eliminate nth harmonic, its amplitude must be zero, i.e., the switching angles must satisfy the following equation

$$[\cos(n\theta_1) + \cos(n\theta_2) + \ldots\ldots + \cos(n\theta_S)] = 0 \ldots (4)$$

For a (2S+1) multilevel inverter having S number of H-bridges, S-1 harmonics can be eliminated. For a single-phase multilevel inverter, the order harmonics which can be eliminated are $3^{rd}$, $5^{th}$, $7^{th}$…etc. depending upon S. For a three-phase multilevel inverter, the order of harmonics that can be eliminated from the output voltage waveform are $5^{th}$, $7^{th}$, $11^{th}$, $13^{th}$ etc. depending upon S as the triplen harmonics are automatically eliminated in line to line voltages.

## III. Forming Trancendental Equations For Different Chmli Inverters

i) 5-level CHMLI Inverter:

For a 5-level cascaded multilevel inverter, the number of H-bridges required is 2 i.e., S=2 (vide Fig. 2).

The output voltage waveform is shown in Fig. 3.

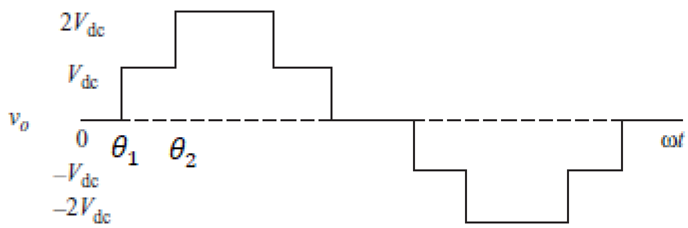


**Fig. 3:** Output voltage waveform of 5-level CHMLI

From equations (3) and (4), the transcendental equations which are to be solved to find out the switching angles are

$$[\cos(\theta_1) + \cos(\theta_2)] = 2M \quad \ldots. (5)$$

and

$$[\cos(3\theta_1) + \cos(3\theta_2)] = 0 \quad \ldots (6)$$

Equation (5) will determine the amplitude of fundamental and equation (6) is used to eliminate 3rd harmonic from the output voltage waveform.

Newton-Raphson iterative method has been used to find out the switching angles by solving the above transcendental equations. The switching angles are shown in Table II for different values of modulation index and different levels of inverter [5].

i) 7-level CHMLI:

For a 7-level cascaded multilevel inverter, the number of H-bridges required is 3 i.e., S=3 (vide Fig. 2).

The output voltage waveform is shown in Fig. 4.

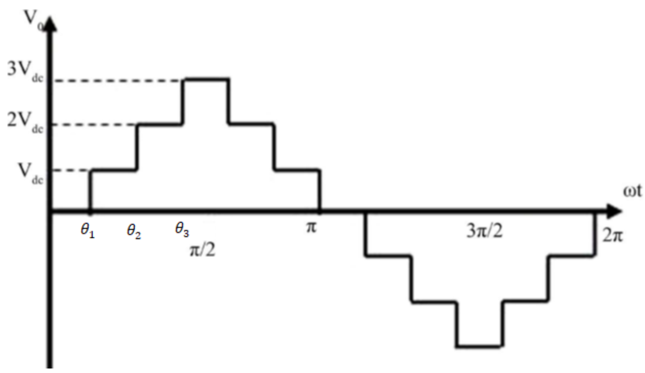


**Fig. 4:** Output voltage waveform of 7-level CHMLI

From equations (3) and (4), the transcendental equations which are to be solved to find out the switching angles are

$$[\cos(\theta_1) + \cos(\theta_2) + \cos(\theta_3)] = 3M \quad \ldots. (7)$$

and

$$[\cos(3\theta_1) + \cos(3\theta_2) + \cos(3\theta_3)] = 0 \quad \ldots (8)$$

$$[\cos(5\theta_1) + \cos(5\theta_2) + \cos(5\theta_3)] = 0 \quad \ldots (9)$$

Equation (7) will determine the amplitude of fundamental and equation (8) and (9) are used to eliminate 3rd and 5th harmonics from the output voltage waveform.

Newton-Raphson iterative method has been used to find out the switching angles by solving the above transcendental equations. The switching angles are shown in Table II for different values of modulation index and different levels of inverter.

i) 9-level CHMLI:

For a 9-level cascaded multilevel inverter, the number of H-bridges required is 4 i.e., S=4 (vide Fig. 2). The output voltage waveform is shown in Fig. 5.

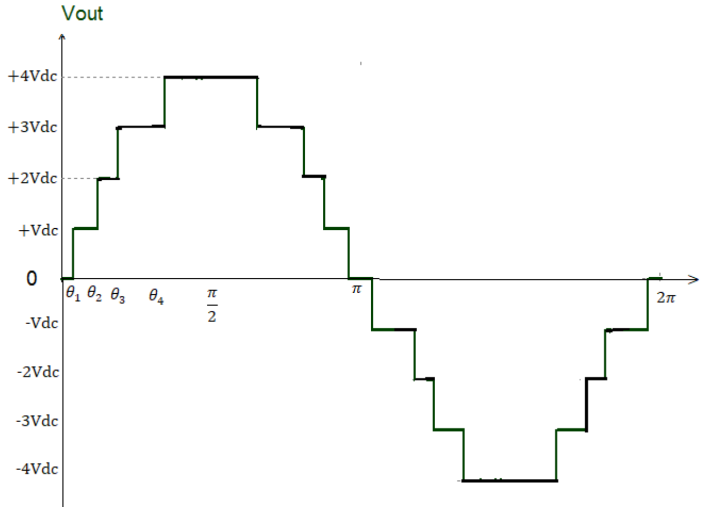


**Fig. 5:** Output voltage waveform of 9-level CHMLI

From equations (3) and (4), the transcendental equations which are to be solved to find out the switching angles are

$$[\cos(\theta_1) + \cos(\theta_2) + \cos(\theta_3) + \cos(\theta_4)] = 4M \quad \ldots (10)$$

and

$$[\cos(3\theta_1) + \cos(3\theta_2) + \cos(3\theta_3) + \cos(3\theta_4)] = 0 \quad \ldots (11)$$

$$[\cos(5\theta_1) + \cos(5\theta_2) + \cos(5\theta_3) + \cos(5\theta_4)] = 0 \quad \ldots (12)$$

$$[\cos(7\theta_1) + \cos(7\theta_2) + \cos(7\theta_3) + \cos(7\theta_4)] = 0 \quad \ldots (13)$$

Equation (10) will determine the amplitude of fundamental and equation (11), (12) and (13) are used to eliminate 3rd, 5th, and 7th harmonics from the output voltage waveform.

Newton-Raphson iterative method has been used to find out the switching angles by solving the above transcendental equations. The switching angles are shown in Table II for different values of modulation index and different levels of inverter.

i) 11-level CHMLI:

For a 9-level cascaded multilevel inverter, the number of H-bridges required is 5 i.e., S= 5 (vide Fig. 2). The output voltage waveform is shown in Fig. 6.

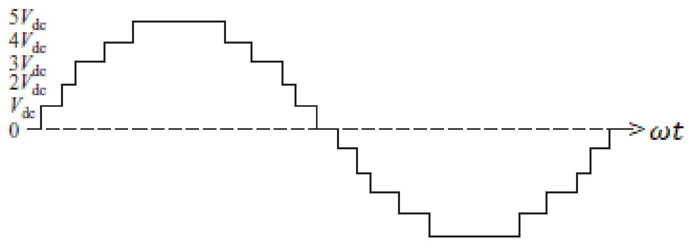


**Fig. 6:** Output voltage waveform of 9-level CHMLI

From equations (3) and (4), the transcendental equations which are to be solved to find out the switching angles are

$$[\cos(\theta_1) + \cos(\theta_2) + \cos(\theta_3) + \cos(\theta_4) + \cos(\theta_5)] = 5M \quad \ldots (14)$$

and

$$[\cos(3\theta_1) + \cos(3\theta_2) + \cos(3\theta_3) + \cos(3\theta_4) + \cos(3\theta_5)] = 0 \quad \ldots (15)$$

$$[\cos(5\theta_1) + \cos(5\theta_2) + \cos(5\theta_3) + \cos(5\theta_4) + \cos(5\theta_5)] = 0 \quad \ldots (16)$$

$$[\cos(7\theta_1) + \cos(7\theta_2) + \cos(7\theta_3) + \cos(7\theta_4) + \cos(7\theta_5)] = 0 \quad \ldots (17)$$

$$[\cos(9\theta_1) + \cos(9\theta_2) + \cos(9\theta_3) + \cos(9\theta_4) + \cos(9\theta_5)] = 0 \quad \ldots (18)$$

Equation (14) will determine the amplitude of fundamental and equation (15), (16), (17) and (18) are used to eliminate $3^{rd}$, $5^{th}$, $7^{th}$, and $9^{th}$ harmonics from the output voltage waveform.

Newton-Raphson iterative method has been used to find out the switching angles by solving the above transcendental equations. The switching angles are shown in Table II for different values of modulation index and different levels of inverter.

**Table II:** Switching angles for different cascaded multilevel inverters (CHMLI) with different modulation index

| Modulation Index, M | Level of Inverter and switching angles in degree | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 5-Level | | 7-Level | | | 9-Level | | | | 11-Level | | | | |
| | $\Theta_1$ | $\Theta_2$ | $\Theta_1$ | $\Theta_2$ | $\Theta_3$ | $\Theta_1$ | $\Theta_2$ | $\Theta_3$ | $\Theta_4$ | $\Theta_1$ | $\Theta_2$ | $\Theta_3$ | $\Theta_4$ | $\Theta_5$ |
| 0.7 | 8.02 | 52.42 | 20.18 | 37.85 | 66.46 | 36.12 | 47.88 | 61.07 | 76.3 | 20.45 | 30.34 | 49.81 | 64.73 | 69.23 |
| 0.8 | 7.2 | 51.9 | 12 | 30.5 | 58.5 | 24.7 | 45.53 | 57.04 | 68.9 | 16.72 | 26.65 | 45.8 | 60.7 | 62.7 |
| 0.9 | 6.53 | 51.2 | 6.23 | 23.74 | 50.67 | 12.44 | 34.59 | 48.81 | 68.89 | 10.21 | 20.65 | 39.73 | 54.66 | 57.92 |

## IV. Simulation Results

Four multilevel inverter circuits similar to Fig. 2 for 5-level, 7-level, 9-level and 11-level have been simulated using MATLAB 2017a simulation software [11] to obtain results of fundamental voltages, harmonic voltages and total harmonic distortion (THD) for each level of multilevel inverters. The simulation circuit for 7-level inverter is shown in Fig. 7(a) and the corresponding output voltage waveform is shown in Fig. 7(b). The result has been shown in Table III. The fundamental frequency and each dc source voltage which have been considered are respectively 50 Hz and 100V.

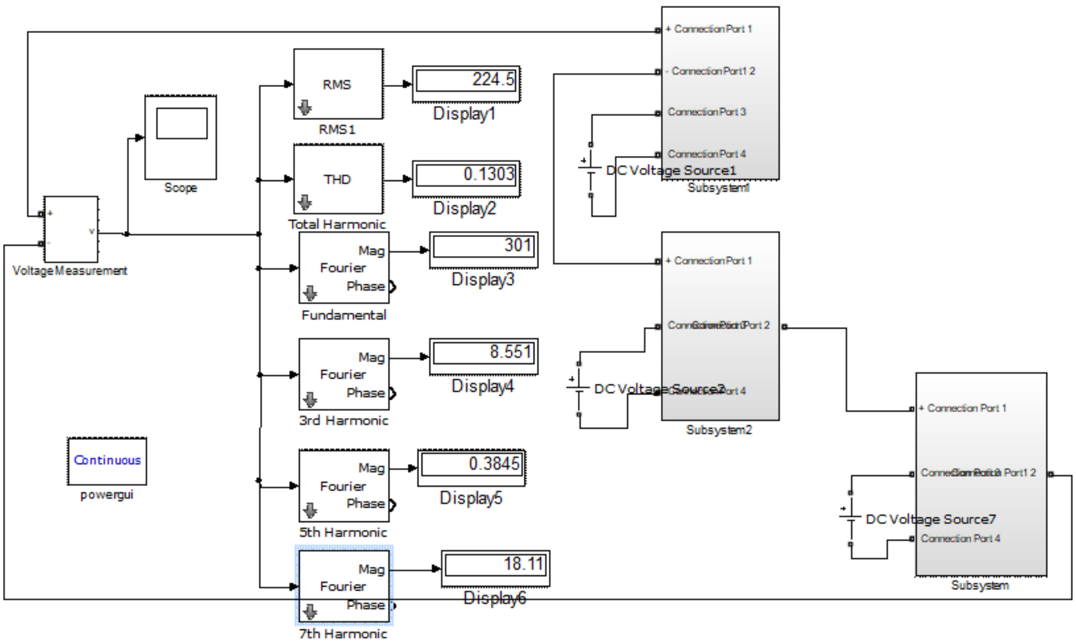


**Fig. 7(a):** 7-level inverter with M=0.8

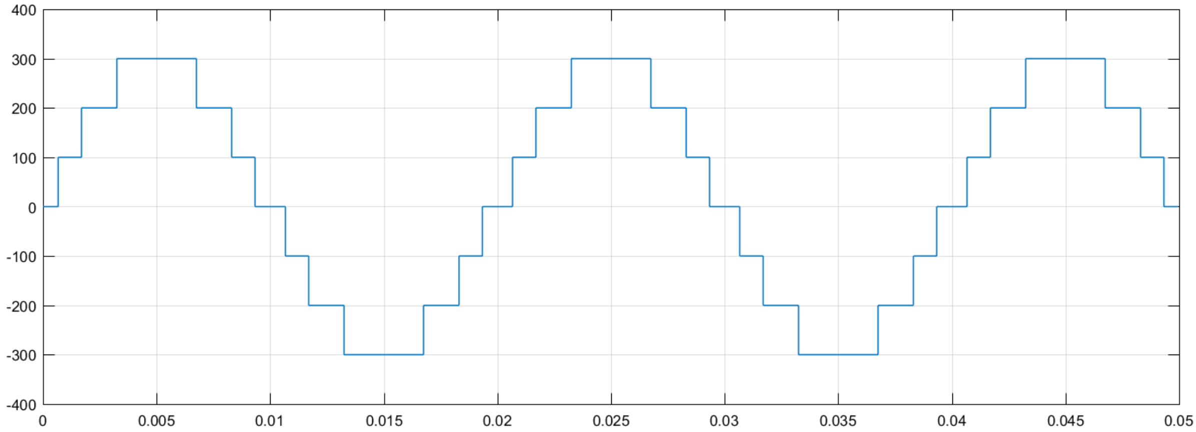


**Fig. 7(b):** Output voltage waveform of 7-level inverter

## V. Conclusion

The paper presents a brief discussion on performance of single-phase four selected levels of CHMLI (5, 7, 9 and 11) with modulation index 0.6, 0.7 and 0.8. The MATLAB simulation was made for elimination of 3rd, 5th, 7th and 9th harmonic voltages from the output voltage waveform (depending upon number of levels) by selecting proper switching angles of the semiconductor devices using Newton-Raphson method. The simulation result shows that the lower order harmonic voltages are comparatively negligible compared to fundamental. except there are some cases where lower order harmonic voltages are present. This is due to selection of switching angles which mainly depends upon selection of initial guess of the switching angles while using Newton-Raphson method as the accuracy of switching angles so obtained using Newton-Raphson method mainly depends upon the initial guesses. The next lower order dominating harmonics are 5th for the 5-level, 7th for the 7-level, 9th for the 9-level and 11th for the 11-level multilevel inverters respectively. These harmonics can be easily filtered out using reduced size low pass filter. From Table III it is seen that the total harmonic distortion (THD) is reduced with the increase of number of levels of inverter as well as modulation index M.

**Table III:** Result showing lower order harmonic voltages for different level of inverters with different modulation index,M

| No of Levels of MLI | Modulation Index | Lower order harmonics as a % of fundamental | | | | THD (%) | Fundamental Amplitude (V) PU |
|---|---|---|---|---|---|---|---|
| | | 3rd Harmonic | 5rd Harmonic | 7rd Harmonic | 9rd Harmonic | | |
| 5 Level | 0.7 | .12 | 8.36 | 14.16 | 0.30 | 20.62 | 101.7 |
| | 0.8 | 0.99 | 6.63 | 14.40 | 2.01 | 20.45 | 103.3 |
| | 0.9 | 1.04 | 6.90 | 14.7 | 2.43 | 20.63 | 103.4 |
| 7 Level | 0.7 | 19.58 | 5.72 | 4.57 | 2.21 | 26.07 | 88.8 |
| | 0.8 | 2.84 | 0.128 | 0.76 | 6.02 | 13.03 | 100.3 |
| | 0.9 | 5.01 | 0.88 | 4.38 | 1.53 | 12.47 | 107.8 |
| 9 Level | 0.7 | 42.56 | 1.21 | 1.0 | 5.06 | 44.33 | 69.6 |
| | 0.8 | 41.6 | 1.09 | 2.6 | 3.74 | 47.4 | 79.62 |
| | 0.9 | 13.86 | 0.60 | 0.41 | 1.21 | 17.63 | 89.65 |
| 11 Level | 0.7 | 23.18 | 2.06 | 5.27 | 6.60 | 26.59 | 82.5 |
| | 0.8 | 18.40 | 0.16 | 0.38 | 8.28 | 22.01 | 88.8 |

## References

### Journal/ Conference Papers:

[1]. A Comparative study of different multilevel inverters_Electronic, IEEE Explore Conference 2017,ISBN 978-1-5386-1703-8, Gautam Ghosh et. el.

[2]. SELECTIVE HARMONIC ELIMINATION OF SINGLE PHASE VOLTAGE SOURCE INVERTER USING ALGEBRAIC HARMONIC ELIMINATION APPROACH, https://www.researchgate.net

[3]. STATIC INVERTER WITH NEUTRALIZATION OF HARMONICS, A. Kernick, J. L.Roof, T. M. Heinrich. *AIEE Transactions,* pt. I I*(Applications and Industry),* vol. 81, May 1962. pp. 59-68

[4]. Selective harmonic elimination in inverters using bio-inspired intelligent algorithms for renewable energy conversion applications: A review Mudasir Ahmed Memona,□, Saad Mekhilefa,□, MarizanMubina,□, Muhammad Aamirb, Renewable and Sustainable Energy Reviews 82 (2018) 2235–2253.

[5]. Solving Nonlinear Equations for Selective HarmonicEliminated PWM Using Predicted Initial Values.

[6]. COMPARISON OF DIFFERENT TECHNIQUES FOR SELECTIVE HARMONIC ELIMINATION PWM IN INVERTERS – A REVIEW, 2017 8th IEEE Annual Information Technology, Electronics and Mobile Communication Conference (IEMCON), ISBN: 978-1-5386-3371-7.

[7]. An Efficient Method for THD Minimization in Multilevel Inverter's Rahim Shamsi Varzeghan, Mohammad Reza JannatiOskuee, Alireza EyvazizadehKhosroshahi, and Sajad Najafi-Ravadanegh, International Journal on Electrical Engineering and Informatics - Volume 8, Number 2, June 2016

[8]. L. M. Tolbert, F. Z. Peng, and T. G. Habetler, "Multilevel converters for large electric drives," IEEE Transactions on Industry Applications, vol. 35, pp. 36—44, Jan./Feb. 1999.

**Books:**

[9]. Muhammad H. Rashid, "Power Electronics".

[10]. Daniel W. Hart, "Power Electronics".

[11]. MATLAB R2017a and Optimization Toolbox, The MathWorks, Inc., Natick, Massachusetts, United States.

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